Influence of entrainment on the thermal stratification in simulations of radiative-convective equilibrium

Supplementary information

Martin S. Singh & Paul A. O'Gorman

S1 CRM simulations

Here we give more details of the CRM simulations described in section 2. Low resolution simulations are first run over a slab ocean to statistical equilibrium with CO_2 concentrations ranging from 1 ppmv to 1280 ppmv (Figure S2). The slab ocean has a depth of 1 m and is assumed to be infinitely conducting such that it has a single temperature over the whole domain. Equilibrium is considered to be reached if the trend in SST over the last 50 days of the simulation is less than 1 K yr^{-1}. The slab-ocean simulations are performed with a horizontal grid-spacing of 2 km and 64 vertical levels in a doubly periodic 80×80 km domain. The time-mean SST of the last 50 days of each slab-ocean simulation is then used as a fixed surface boundary condition for higher resolution simulations (1 km horizontal grid spacing, 64 vertical levels, 84×84 km domain) with the corresponding CO_2 values. These simulations equilibrate much faster because the SST is fixed and because the simulations are initialized with the mean state of the slab-ocean simulations. The total atmospheric energy and water vapor reach their equilibrium values in 10 - 20 days (Figure S1). The fixed-SST simulations are run for 40 days; statistics collected over the last 20 days are used for the results in the paper. A subset of the simulations were also performed using a horizontal grid spacing of 0.5 km, and the results are found to be insensitive to horizontal grid spacing over the range 0.5 km – 2 km. Ozone mixing ratio is fixed in all simulations to a standard profile with a maximum at around 10 hPa.

The slab ocean simulations at the two highest SSTs are affected by a form of convective self-aggregation which tends to decrease SST. The corresponding fixed-SST simulations do not show evidence of aggregation, and our results are not expected to be substantially affected by this issue.

S2 Description of zero-buoyancy entraining plume model

In this section we give a detailed description of the zero buoyancy plume model used in sections 3.1 and 4. We consider a steady entraining plume as a bulk representation of an ensemble of cumulus clouds (e.g., *Tiedtke*, 1989). Neglecting kinetic energy and assuming hydrostasy, we may write the governing equations for the plume in terms of the conserved variables (in the absence of precipitation fallout) moist-static energy h and total water mass fraction q_t as

$$\frac{dh}{dz} = -\epsilon(h - h_e), \tag{1}$$

$$\frac{dq_t}{dz} = -\epsilon\left(q_t - q_{ve}\right). \tag{2}$$

Here z is height, ϵ is the entrainment rate (in units of inverse length), the subscript e refers to the environment, and h and q_t are expressed per unit total mass of moist air. The moist static energy is

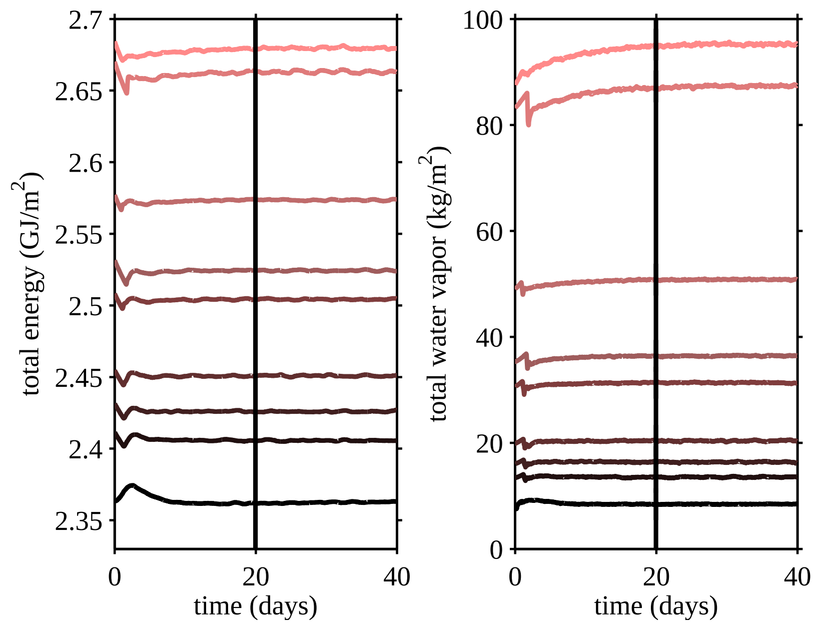

Figure S1: Timeseries of total atmospheric energy (left) and water vapor (right) for fixed-SST simulations with the CRM. The vertical black line corresponds to the time at which the collection of statistics begins.

defined as

$$
\begin{aligned}
h &= (1-q_t)h_d + q_v h_v + q_l h_l + q_i h_i, \\
h_d &= c_{pd}(T-T_0) + gz, \\
h_v &= c_{pv}(T-T_0) + gz + L_{v0}, \\
h_l &= c_l(T-T_0) + gz, \\
h_i &= c_i(T-T_0) + gz - L_{f0},
\end{aligned}
$$

where T is the temperature, g is the gravitational acceleration, and the constants L_{v0} and L_{f0} are the latent heats of vaporization and fusion, respectively, at the reference temperature T_0. The total water mass fraction q_t has components associated with vapor (q_v), liquid (q_l), and ice (q_i). The isobaric specific heat capacity of dry air and water vapor are given by c_{pd} and c_{pv}, while liquid water and ice are treated as incompressible, with specific heat capacities c_l and c_i respectively. All constants are taken as the values used in the CRM and are given in table S1.

Equations (1) and (2) assume no fallout of precipitation, and this form of the zero-buoyancy plume model is used when considering reversible CAPE (Figures S4 and S5). However, the default version of the zero-buoyancy plume model assumes complete fallout of all liquid and solid water from the plume. Fallout is applied in a separate step in which all liquid and solid water are removed, and the moist static energy is recalculated at constant pressure and temperature.

As in the CRM, the saturation vapor pressure e^* is written as a combination of the saturation vapor pressure over liquid (e_l^*) and ice (e_i^*) according to

$$
e^*(T) = \zeta(T)e_l^*(T) + (1-\zeta(T))e_i^*(T),
$$

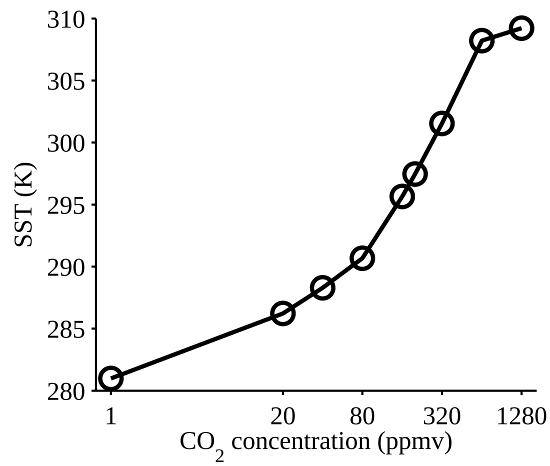

Figure S2: Equilibrium SSTs in CRM simulations of RCE with a slab ocean and varying CO_2 concentration. CO_2 concentration is shown on a log scale.

where $\zeta(T)$ increases linearly in temperature from zero to one between 233.15 K and 273.15 K. At saturation, the phases of water are divided according to

$$\begin{aligned} q_v &= q_v^*, \\ q_l &= \zeta(q_t - q_v^*), \\ q_i &= (1-\zeta)(q_t - q_v^*), \end{aligned}$$

where q_v^* is the saturation specific humidity.

The plume model is solved in the zero-buoyancy limit in which the plume is exactly neutrally buoyant with respect to the environment. [In this limit (1) and (2) may also be obtained by applying the parcel model of *Romps and Kuang* (2010) to a steady, vertically oriented flow with no precipitation fallout.] The inputs are the entrainment rate profile, the temperature and specific humidity near the surface (or at 850hPa in section 4), and a nominal value of the environmental relative humidity. Equations (1) and (2) are integrated upwards to solve for the environmental temperature. The integration is performed using forward differences with a step size of 50 m. At each step, the temperature and humidity of the plume are calculated from h and q_t. If fallout is specified, the liquid and solid water are removed as described above. The environmental temperature is then calculated by assuming the density of the plume and environment are equal and using the environmental relative humidity as given.

References

Romps, D. M., and Z. Kuang (2010), Do undiluted convective plumes exist in the upper tropical troposphere?, *J. Atmos. Sci.*, *67*, 468–484.

Tiedtke, M. (1989), A comprehensive mass flux scheme for cumulus parameterization in large-scale models, *Mon. Wea. Rev*, *117*, 1779–1800.

Table S1: Thermodynamic constants used in the CRM and zero-buoyancy plume model.

Symbol	Value	Units
g	9.81	m s^{-2}
c_p	1005.7	J kg^{-1} K^{-1}
c_{pv}	1870.0	J kg^{-1} K^{-1}
c_l	4190.0	J kg^{-1} K^{-1}
c_i	2106.0	J kg^{-1} K^{-1}
R_d	287.04	J kg^{-1} K^{-1}
R_v	461.50	J kg^{-1} K^{-1}
L_{v0}	2.501×10^6	J kg^{-1}
L_{f0}	2.069×10^5	J kg^{-1}
T_0	273.15	K

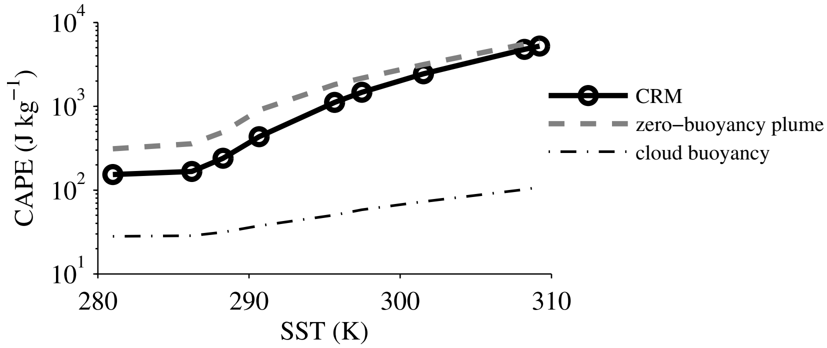

Figure S3: As in Figure 1a, but plotted with a logarithmic scale for the ordinate.

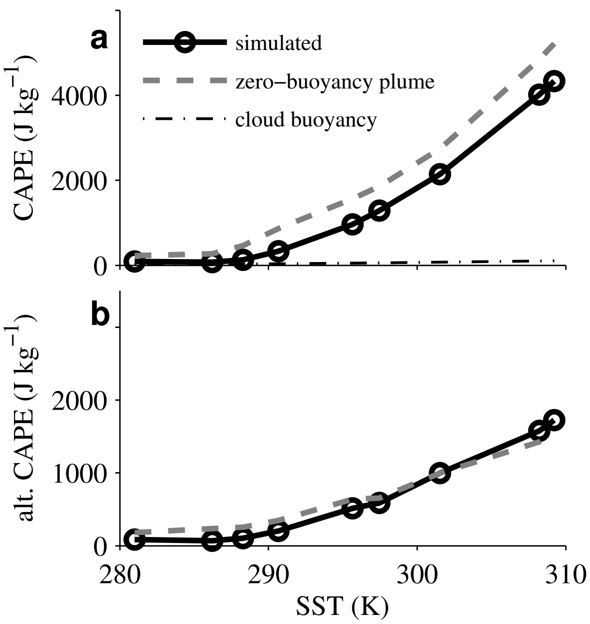

Figure S4: As in Figure 1, but for reversible CAPE, no precipitation fallout in the zero-buoyancy plume model, and an entrainment parameter of $\hat{\epsilon} = 0.5$.

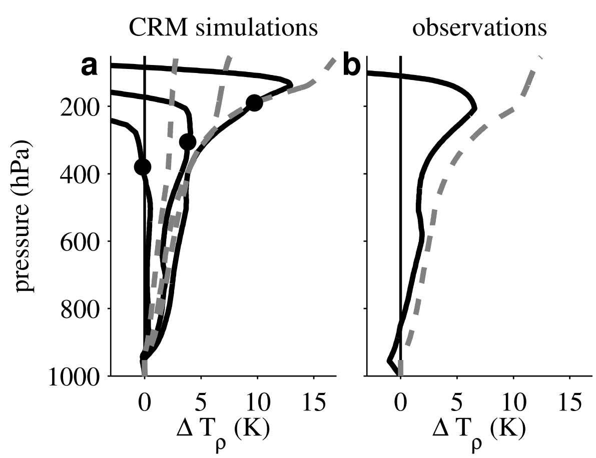

Figure S5: As in Figure 2, but for a reversible parcel ascent, no precipitation fallout in the zero-buoyancy plume model, and an entrainment parameter of $\hat{\epsilon} = 0.5$. Density temperature difference, ΔT_ρ, is shown rather than virtual temperature difference because the reversible parcel contains some condensate. The subset of soundings used in (b) is the same as that used for figure 2b.