12.812: Problem set 1

Due in class on October 3rd, 2019

Need help?: Office hours Wednesday 11am-noon; Also email or drop by as needed.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

The purpose of this first problem set is to give you practice using the zonal and time eddy/mean decomposition, using the sigma coordinate, and assessing large-scale data from reanalysis.

1. **Space and time decomposition into eddies and means**: In class, we discussed how variables may be split into means and eddies with respect to both a time and zonal mean. We decompose a variable A into a time mean and eddy as

$$A = \overline{A} + A',$$

and into a zonal mean and eddy as

$$A = [A] + A^*.$$

(a) Beginning with these expressions, show that the time and zonal mean meridional temperature flux $[\overline{vT}]$ may be decomposed as follows:

$$[\overline{vT}] = [\overline{v}][\overline{T}] + \overline{[v]'[T]'} + \overline{[v^*T^*]}.$$

(b) Explain in words what each of the three contributing terms are in this decomposition of the temperature flux, and give examples of the corresponding circulations in the atmosphere in each case.

2. **Vertical coordinate systems**: The sigma vertical coordinate is widely used in modeling and analysis of the atmospheric general circulation. The sigma coordinate is defined by $p = \sigma p_s$ where p is pressure and p_s is surface pressure. For the (x, y, σ) coordinate system, and working within the hydrostatic approximation, show that the continuity equation is given by

$$\frac{\partial p_s}{\partial t} + \nabla_\sigma \cdot (p_s \mathbf{v}) + \frac{\partial (p_s \dot{\sigma})}{\partial \sigma} = 0,$$

where $\dot{\sigma} = D\sigma/Dt$ is the Lagrangian rate of change of σ and $\mathbf{v} = (u, v)$ is the horizontal fluid velocity. The simplest approach is to consider the conservation of mass in an infinitesimal fluid element following the flow, but it is also acceptable to take the continuity equation in the (x, y, p) coordinate system as a starting point.

3. **Zonal asymmetry of the relative humidity distribution**

(a) Plot a latitude-longitude cross-section of the mean relative humidity at 500hPa for June-July-August using reanalysis data averaged between 1990 and 2010. It is convenient to use the interactive plotting website at www.esrl.noaa.gov/psd/cgi-bin/data/composites/printpage.pl to make the cross-section. You will likely want to change the contour interval.

(b) Now plot the mean vertical velocity ω at 500hPa for the same period and comment on how it compares to the relative humidity cross-section. Explain how vertical motion affects relative humidity, using equations where necessary.

(c) Test whether mean subsidence alone can explain the humidity in the subsidence regions at 500hPa by comparing the specific humidity distributions at 300hPa and 500hPa. Explain your reasoning. What other processes, besides mean subsidence, could be contributing to the maintenance of the mean humidity in the mean subsidence regions?